ARTICLE

The Political Economy of Social Networks: Analyzing Friendship Patterns among University Students Using Graph Theory

Adams Amadu Diiwu[1] | David Akanmike Akanluke[2] | Emmanuel Agyapong[2] | Jude Nii Nai Johnson[2] | Peniel Buaka[2]

[1] School of Metamathematics, Nile Valley Multiversity, Techiman, Ghana. Email: amaduadams49@gmail.com
[2] Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana

Abstract

This study examines the friendship patterns of final-year mathematics students at the Kwame Nkrumah University of Science and Technology (KNUST) by analyzing their social media connections across different departments. Using graph theory and algorithms, the study models students and their friendships as a graph, which is then analyzed to identify patterns of departmental affiliations. The Breadth First Search (BFS) algorithm was employed to conduct the analysis. The findings reveal that final-year mathematics students had a higher frequency of friends in the actuarial science department than in other departments. This outcome can be attributed to the overlap between the courses and classes taken by students in both mathematics and actuarial science, fostering frequent interactions and stronger relationships. These interactions contribute to the creation of academic and social ties between students of these two disciplines. From a political economy perspective, this study highlights the importance of social networks in shaping academic and career opportunities. The closeness between mathematics and actuarial science students may facilitate the sharing of academic resources, including internship opportunities and study materials. Such exchanges are crucial for career advancement in fields like actuarial science, where practical experience is highly valued. However, the relatively limited social interactions with students from other departments may restrict the breadth of students' academic and professional networks, potentially limiting their exposure to diverse perspectives and opportunities. This study underscores the implications of social media networks in higher education, where relationships between students within the same or related fields can impact not only their academic success but also their future professional networks. Expanding these networks beyond departmental boundaries may offer a wider array of opportunities, both socially and professionally, contributing to a more holistic academic experience.

KEYWORDS:
Graph theory; social media; Breadth first search algorithm; political economy; KNUST

1 | INTRODUCTION

Graph theory, the study of mathematical structures used to model pairwise relationships between objects, plays a critical role in analyzing complex networks. In graph theory, relationships between entities are represented as edges (lines), while the entities themselves are nodes (points). This mathematical framework is widely applicable across many domains, from computer science to economics and biology. For instance, in computer science, graph theory is used to model networks, such as the internet and social media platforms, where nodes represent users and edges represent relationships or interactions between them (Newman, 2010). In economics, graphs are instrumental in illustrating income distribution, such as through the Lorenz curve, which shows wealth inequality (Lorenz, 1905). Additionally, in biology, molecular structures of cells and genes are often modeled as graphs to understand the intricate relationships between biological components (Barabási, 2002).

The application of graph theory has grown significantly in the realm of social networks, particularly with the rise of social media platforms like Facebook, WhatsApp, and X. These platforms have transformed how individuals form relationships, share information, and influence one another. According to Data Reportal (2024), the typical internet user spends nearly seven hours online daily, with a significant portion of this time spent on social media—approximately 2 hours and 27 minutes per day. Social media usage has been shown to shape personal and professional networks, particularly among university students, who often form and maintain relationships through these platforms (Ellison et al., 2007). For instance, Facebook, in particular, allows students to create networks that transcend the physical boundaries of university campuses, leading to the formation of diverse social circles that can influence their academic and career trajectories (Lampe et al., 2006).

This study leverages graph theory to explore the dynamics of Facebook friendships among final-year mathematics students at the Kwame Nkrumah University of Science and Technology (KNUST). Specifically, it investigates how students connect with peers from other departments and the implications of these relationships. The analysis utilizes algorithms from graph theory to model the networks formed by these students and identifies patterns in their friendships across various departments. In particular, the Breadth First Search (BFS) algorithm, commonly used in network analysis (Cormen et al., 2009), is employed to detect the structural features of these social connections.

The political economy of these social networks is crucial for understanding how access to resources and opportunities is distributed among students. While social media interactions within academic disciplines, such as mathematics and actuarial science, can provide academic and professional advantages through shared resources and opportunities (Feld, 1981), the lack of cross-departmental connections may limit students' exposure to broader perspectives. This dynamic reflects how social and academic networks, shaped by digital platforms, contribute to the formation of power structures within educational institutions. In the context of the global political economy, the ability to navigate and leverage these networks can play a significant role in determining future career prospects, access to information, and ultimately economic outcomes (Bourdieu, 1986).

In sum, this study illustrates the power of graph theory in understanding social networks and their implications for students' social cohesion and academic success. By examining how final-year mathematics students interact with peers from different departments, the study not only reveals important patterns in their social networks but also highlights how these networks can influence broader political and economic structures, shaping future opportunities and outcomes for individuals in the academic and professional spheres.

2 | LITERATURE REVIEW

The literature review examines already existing research on graph theory and its applications in social network analysis. It discusses algorithms like Breadth First Search (BFS), Depth First Search (DFS), and various centrality measures, offering a comparative analysis of their strengths and weaknesses. This chapter establishes a theoretical basis for the methodology used in the article by exploring how these algorithms have been applied in related studies. Graph Theory plays an important role in analysing social networks. Several algorithms can be deployed to aid in the analysis. Some of these algorithms include Breadth First Search algorithm, Depth Search algorithm, The Bellman-Ford algorithm, Degree Centrality and the Dijkstra algorithm. In this work, the Breadth First Search method was employed to analyse the friends of final year mathematics students' connections on Facebook.

The Breadth First Search method is a graph traversal algorithm that explores all nodes in a graph at the current depth before moving on to the next node at the next depth level. In this study, the power of Breadth-First Search (BFS) was harnessed to unveil the intricate web of Facebook friendships. The aim was to delve deep into the structure of social connections among users. By utilizing this algorithm, we embarked on an expedition through the social network graph, commencing from a specific student and intricately exploring their connections. This approach enabled us to unearth clusters of tightly knit friendships, discern which departments or programs were most frequently linked with Final year mathematics students, and lay bare the overall interconnectedness of the social network.

Despite the usefulness of Breadth First Search (BFS) method, it is not without limitations. One of the key challenges in using BFS for network analysis is the bias it introduces, particularly its tendency to oversample high-degree nodes those individuals in the network who have many connections (Kurant et al., 2010). In the context of Facebook friendship analysis, this means that BFS might disproportionately highlight popular users with many friends, while underrepresenting users with fewer connections. This could lead to a skewed understanding of the social network, where certain groups or individuals appear more central or influential than they actually are. It has been empirically observed that incomplete BFS is biased toward high degree nodes. Understanding this bias is crucial for accurately interpreting the results of our analysis. For example, if BFS suggests that Final year mathematics students are predominantly connected to students from a particular department, it is important to consider whether this result might be influenced by the presence of highly connected individuals within that department, rather than a true reflection of widespread interaction. To mitigate these biases, the insights from the paper can guide the application of correction techniques or the adoption of complementary sampling methods in the study. By acknowledging and adjusting the bias, the analysis of Facebook friendships using BFS can provide a more accurate representation of the social interactions among Final year mathematics students, leading to more valid conclusions about their social network structure. Thus, linking the findings of Kurant et al (2010) to this work enhances the robustness of the analysis by highlighting potential pitfalls and suggesting strategies to ensure that the use of BFS in exploring Facebook friendships yields reliable and meaningful insights.

The research conducted by Zhou and Hansen (2006) on Breadth-First Heuristic Search (BFHS) offers a possible method to enhance the BFS procedure. Heuristics are problem-specific strategies that BFHS can use to prioritize nodes or paths in the graph that are most likely to yield significant insights. Heuristics, for instance, could direct BFS to investigate specific regions of the social network more effectively if specific patterns of friendship connections are known or predicted, possibly revealing significant linkages more quickly than with regular BFS. A heuristic method could be utilized in the case of examining Facebook friendships among Final year mathematics students in order to narrow down the search to connections involving students who are well-known for being very active on social media or who have a track record of developing friendships across departmental boundaries. This would assist in identifying important influencers or bridge nodes within the network in addition to increasing the efficacy of the study by eliminating pointless investigation. Furthermore, the research by Zhou and Hansen (2006) discusses the incorporation of heuristics, which may help with some of the difficulties caused by the enormous size and complexity of social networks on sites like Facebook. As this study involves many nodes (students) and edges (friendships), a heuristic BFS could significantly improve the performance and scalability of the analysis, ensuring that meaningful insights are derived without excessive computational cost. The concepts from the "Breadth-First Heuristic Search" can enhance this work by offering a pure version of BFS that is better suited for large-scale social network analysis. Building on these strategies leads to more efficient exploration of the Final year mathematics students' social network, uncovering patterns and relationships that might be overlooked by a standard BFS approach.

The investigation, which applies graph theory and BFS to examine Final year mathematics students' Facebook friendships, is strongly related to the topics discussed in Sihotang's (2020) paper. Examining the student social network can be closely linked to using BFS to find the shortest paths in a graph. Finding the shortest path using BFS within the context of this research may be very helpful for figuring out how closely connected various students or groups of students are to one another within the network. BFS, for instance, can be used to determine the shortest path of connection between two students who are connected by mutual friends but are not friends directly. This analysis highlights the social network's cohesiveness and strength and point out important players who serve as links between various groups. In addition, using BFS to locate shortest pathways can assist in determining the most effective channels for influence or communication within the social network. For example, examining the shortest connections between important nodes (students) could reveal important information about how rapidly trends or knowledge could spread across final year mathematics pupils. The usefulness of BFS in unweighted graphs is highlighted in Sihotang's (2020) study, which is especially pertinent to social networks since friendships—the edges—typically lack weights. Because it eliminates the need for edge weights and lets one concentrate on the pure connectivity between students, BFS is the perfect tool for analysis. Sihotang's (2020) article provides valuable insights that can bolster the methodological foundation of this work. Specifically, it highlights the efficacy of BFS in social network structure analysis, notably in identifying shortest paths between individuals. This strategy not only supports the goals of the study but also improves comprehension of the connections between Final year mathematics students on Facebook.

Cao et al (2011) concludes that search for shortest paths is an essential primitive for a variety of graph-based applications, particularly those on online social networks. For example, LinkedIn users perform queries to find the shortest path "social links" connecting them to a particular user to facilitate introductions. This type of graph query is challenging for moderately sized graphs but becomes computationally intractable for graphs underlying today's social networks, most of which contain millions of nodes and billions of edges. They propose Atlas, a novel approach to scalable approximate shortest paths between graph nodes using a collection of spanning trees. Spanning trees are easy to generate, compact relative to original graphs, and can be distributed across machines to parallelize queries. They demonstrate its scalability and effectiveness using 6 large social graphs from Facebook, Orkut and Renren, the largest of which includes 43 million nodes and 1 billion edges. They describe techniques to incrementally update Atlas as social graphs change over time. They capture graph dynamics using 35 daily snapshots of a Facebook network and show that Atlas can amortize the cost of tree updates over time. Finally, they apply Atlas to several graph applications and show that they produce results that closely approximate ideal results.

Wadhwa (2000) in his work research targeted a Network Design Problem (Cable and Trench Problem), which involves a trade-off between utilization costs and capital costs for network construction. A larger network, (the shortest path tree) may cost more to build but may reduce utilization costs by including more attractive origin-destination paths. Conversely, a smaller network, (minimum spanning tree) may increase the utilization costs. A heuristic has been provided which gives us optimal or near optimal solutions. This heuristic is an adaptation of the Savings algorithm given by Clarke and Wright in 1964, for solving a vehicle routing problem. The heuristic provides us good solutions which can be used as upper bounds for branch and bound methods, giving us the optimal solutions in lesser times than that given by branch and bound without the upper bounds.

Goyal (2010) studied Travelling Salesman Problem (TSP) problem in combinatorial optimization in both operations research and theoretical computer science. Given a list of cities and their pair wise distances, the task was to find a shortest possible tour that visits each city exactly once. It was first formulated as a mathematical problem in 1930 and is one of the most intensively studied problems in optimization. Problems having the TSP structure most commonly occur in the analysis of the structure of crystals, in material handling in a warehouse, the clustering of data arrays. Related variations on the traveling salesman problem include the resource constrained traveling salesman problem which has applications in scheduling with an aggregate deadline. The prize collecting traveling salesman problem and the orienteering problem are also special cases of the resource constrained TSP. Most importantly, the traveling salesman problem often comes up as a sub problem in more complex combinatorial problems, the best known and important one of which is the vehicle routing problem. That is, the problem of determining for a fleet of vehicles, which customers should be served by each vehicle and in what order each vehicle should visit the customers assigned to it. Due to the nature of TSP, most common solutions to the problem were found to run feasibly only for a graph with small number of nodes. Not much research was encountered in the survey over problem space analysis of the Travelling Salesman problem. However common approaches have been found to possess a critical region roughly around n = 40[14], after which they start taking increasingly even more time to halt. This happens due to their inefficient time growth functions with respect to the number of nodes. Concorde a very well known, fast and exact solution, however, holds a record to have solved for 15,112 cities as well. In this research author refer Warshall's Algorithm.

3 | METHODOLOGY

This section details the methodological approach of the study, focusing on the application of graph theory to map out and analyse the Facebook friendships of final year mathematics students. It describes the data collection process, the construction of social network graphs, and the specific use of BFS to explore these networks. The section also discusses the types of graphs used, including directed, undirected, weighted, and bipartite graphs, and the visualization techniques applied to represent the network. A graph represents a network which consists of a set of objects, mathematically called vertices or nodes. These vertices or nodes are interconnected to one another based on some relation, through edges or arcs. Simply put, a graph is a finite number of points (known as nodes or vertices) connected by lines (known as edges or arcs).

Vertices or **Nodes** are the fundamental units of which graphs are formed. That is, points within a graph that are connected with lines. These represent the students in various departments. **Edges** are line segments used to interconnect vertices or nodes. Per this study, an edge is the relationship between two friends on Facebook. Based on the kind of edges, graphs can be classified into directed and undirected graphs.

a) Directed graph: Here the arcs between two vertices have a particular direction; it is directed from one vertex to another with the direction usually represented by an arrow.

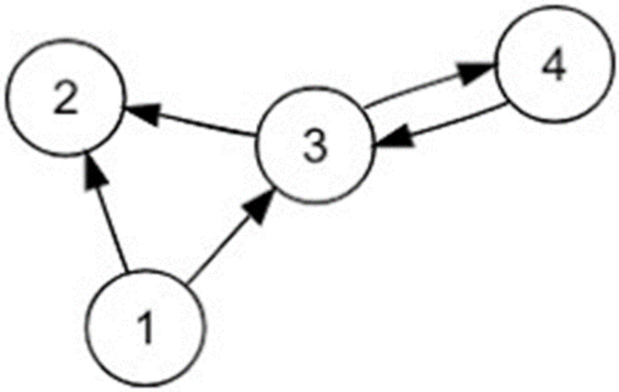

FIGURE 1 Example of a directed graph

b) Undirected graph: Here the edges between two vertices have no particular direction.

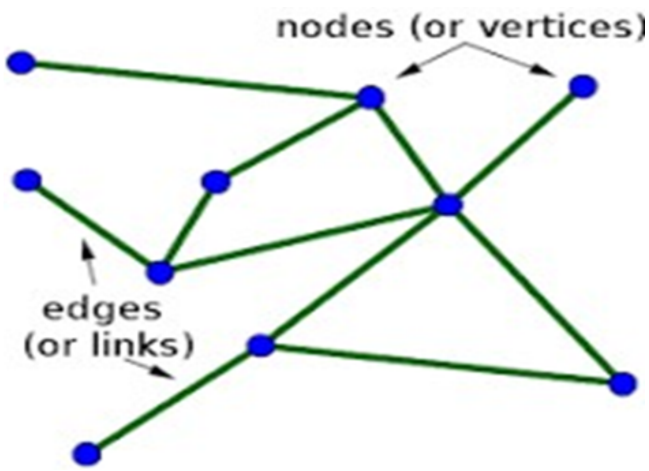

FIGURE 2 Example of an undirected graph

Besides the kind of edges, a graph can also be classified based on the presence or absence of cycles formed in the graph. This gives rise to rise to cyclic and acyclic graphs.

a) Cyclic Graph: If a cycle is formed in a graph, then it is called a cyclic graph. It is a closed figure which starts and ends with the same vertex.

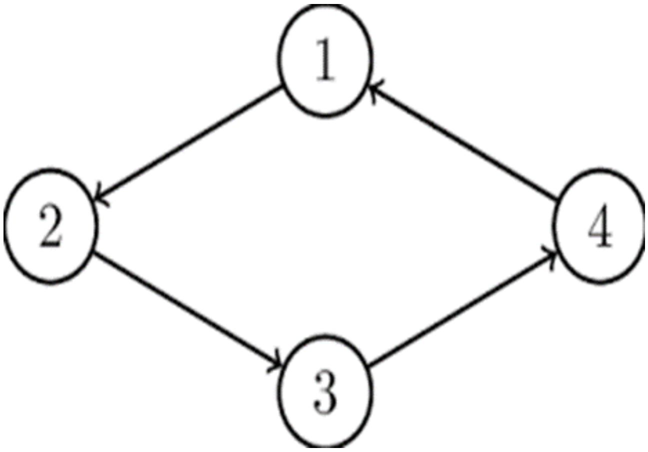

FIGURE 3 Example of a cyclic graph

b) Acyclic Graph: If the graph forms no cycle, then it is called an acyclic graph.

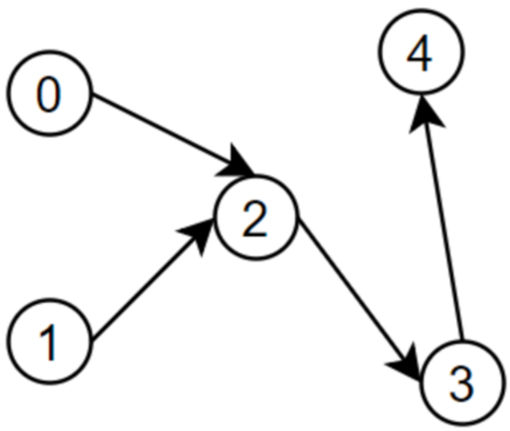

FIGURE 4 Example of an acyclic graph

A graph G is called a tree if it is connected and has no cycles. That is, a tree is a connected acyclic (circuitless) graph. A graph is a tree if and only if there is exactly one path between every pair of its vertices. A graph G is said to be a bipartite graph if its vertex set V can be partitioned into two sets, say V1 and V2, such that no two vertices in the same partition can be adjacent. Here, the pair (V1,V2) is called the bipartition of G. A graph, G is termed weighted if each edge of G, e has been assigned a real number w(e) called the weight or length. An unweighted graph, G on the other hand is a graph in which the edges, e have no weights w(e).

3.1 | Breadth First Search (BFS)

Breadth First Search (BFS) is a graph traversal algorithm that explores all the vertices in a graph at the current depth before moving on to the vertices at the next depth level. It starts at a specified vertex and visits all its neighbours before moving on to the next level of neighbours. BFS is commonly used in algorithms for pathfinding, connected components, and shortest path problems in graphs. BFS systematically explores the edges of a graph, G to "discover" every vertex that is reachable from the source vertex(s). It computes the distance (smallest number of edges) from beginning vertex(s) to each reachable vertex. It also produces a "breadth-first tree" with roots that contain all reachable vertices. For any vertex v reachable from s, the simple path in the breadth-first tree from s to v corresponds to a "shortest path" from s to v in G, that is, a path containing the smallest number of edges. The algorithm works on both directed and undirected graphs. Breadth First Search is so named because it expands the frontier between discovered and undiscovered vertices uniformly across the breadth of the frontier. That is, the algorithm discovers all vertices at distance k from s before discovering any vertices at distance k +1.

Advantages of Breadth First Search (BFS)

i. BFS is very simple to implement since it explores all adjacent nodes before taking the next vertex which makes the algorithm easy to understand.

ii. BFS is often used in broadcasting algorithms where messages need to be sent to all vertices in a network in the least number of hops.

iii. In an unweighted graph, BFS determines the shortest path between any two nodes. This is due to the fact that BFS investigates every node at the current depth level before going on to the next depth level aside exploring and displaying all the nodes in the graph.

Disadvantages of Breadth First Search (BFS)

i. When determining the shortest path, BFS performs poorly on graphs with weighted edges. Algorithms like Dijkstra's or A* are more suited for these situation.

ii. When dealing with extremely large graphs, BFS's performance may be impacted by its requirement to investigate every node at a given level before going on to the next, which may require a lot of time and resources.

iii. If finding a certain node or path quickly is the goal, BFS might not always be the most efficient algorithm when compared to other options like Depth-First Search (DFS).

3.2 | Data collection

The study was conducted to analyse the social networks of Final year mathematics students on Facebook. Thus, to find out the program final year mathematics pupils usually befriend, taking Facebook into account. In doing this, data was collected by finding out the Facebook friends of final year mathematics students. It proved quite challenging trying to extract the friends of various final year mathematics students on Facebook as well as their respective programs. The vast majority of people either do not include their academic profile, including courses of study on Facebook or user names that makes it difficult to identify them on Facebook. This proved to be a huge challenge in the quest for obtaining the data required for the work at hand. We then decided to do the data collection manually, by taking the names of the Final year mathematics students alongside their various friends and respective programs. To conduct a comprehensive and meaningful analysis, it was crucial to have access to reliable and consistent data. Hence the selection was done arbitrarily from friends. This group was then treated as a sample, seeing as it would have been tedious trying to get the data of the entire class of over 170 pupils. Also, not everyone in the class had an active Facebook account during the time of the data collection. The diversity of friends enhances the generalizability of the results and allows for a more comprehensive understanding of the relationship between final year mathematics students and their friends. The data collected was used to generate a graph where the search algorithm was applied for proper analysis. The collected data was used to construct a graph where nodes represent students and edges represent friendships among the students. The BFS path was analysed to determine the frequency of different academic programs among the friends of the mathematics students. The table of the collected data of final year students and their friends in other departments with their respective courses is shown in Table 1.

TABLE 1 Final-Year Mathematics Students and Their Peers' Programs

MATH 4 STUDENTS	FRIENDS IN OTHER DEPARTMENTS AND RESPECTIVE PROGRAMS
AL	AAM_ECONOMICS (EC), WM_ACTUARIALSCIENCE (AS), LB_FRENCH (FC), GA_BIOLOGICAL SCIENCE (BS) BL_ACTUARIALSCIENCE (AS)
ADA	AM_ACTUARIALSCIENCE (AS), SA_PHYSIOTHERAPY (PHYT), AC_STATISTICS (ST), LKK_STATISTICS (ST), BD_BUSINESS ADMINISTRATION (BA)
DKD	NE_QUANTITYSURVEYING (QS), BE_ACTUARIALSCIENCE (AS), AI_STATISTICS (ST), AP_GEOLOGICALENGINEERING (GE), JAM_ELECTRICAL ENGINEERING (EEG)
ND	AZ_BUSINESS ADMINISTRATION (BA), ON_PHYSICS (PY), HD_COMPUTERSCIENCE(CS), KS_ACTUARIALSCIENCE (AS), KJ_STATISTICS (ST)
GSA	SM_MECHANICALENGINEERING (ME), AC_ENVIRONMENTALSCIENCE (ES), AO_ACTUARIALSCIENCE (AS), SO_OPTOMETRY (OP), AF_PETROLEUMENGINEERING (PE)
NR	IT_ELECTRICAL ENGINEERING (EEG), AC_ACTUARIALSCIENCE (AS), JD_HISTORY (HIS), SN_AGRICULTURE (AG), CE_ACTUARIALSCIENCE (AS)
RF	ML_NURSING (NUR), RI_ACTUARIAL SCIENCE (AS), KM_HISTORY (HIS), NBS_OPTOMETRY (OPT), NM_ACTUARIALSCIENCE (AS)
ZK	OAK_NATURALRESOURCEMANAGEMENT (NRM), AP_NURSING (NUR), BV_CHEMISTRY (CHEM), EB_ECONOMICS (EC), LT_ACTUARIAL SCIENCE (AS)
AF	KN_BUSINESS ADMINISTRATION (BA), MA_ACTUARIALSCIENCE (AS), KG_SOCIOLOGY (SOCIO), BKB_BIOCHEMISTRY (BC), GK_PHARMACY (PHA)
FA	BE_BIOLOGICAL SCIENCE (BS), AA_HUMANBIOLOGY (HB), BG_COMPUTERSCIENCE (CS), KA_GEOLOGICALENGINEERING (GEOENG), AOP_ACTUARIALSCIENCE (AS)
AI	AK_TELECOMENGINEERING (TELENG), AB_MEDIACOMMUNICATION (MEDIACOM), DA_ARCHITECTURE (ARCHI), EB_BUSINESS ADMINISTRATION (BA), OAA_ACTUARIALSCIENCE (AS)
NB	AB_BUSINESS ADMINISTRATION (BA), AA_GEOLOGICALENGINEERING (GEOENG), SO_AGRIBUSINESS MANAGEMENT(AGB), HA_STATISTICS (ST), BI_ACTUARIAL SCIENCE (AS)

Table 1 in the article presents data on the friendships of final-year mathematics students with peers from other academic programs at KNUST. The analysis reveals that actuarial science (AS) students form the largest share of their social connections. This is likely due to the significant overlap in coursework, shared classes, and academic collaborations between the two disciplines, which create opportunities for social interaction and study partnerships. The second most frequent group is business administration (BA) students, which suggests a potential networking interest among mathematics students in finance, banking, and investment-related careers.

Surprisingly, statistics (ST) students, despite their academic similarity to mathematics, have fewer recorded friendships than actuarial science students. This may be due to differences in course structures, class schedules, or even departmental culture, which may limit interaction. Additionally, the presence of students from other fields such as engineering, physics, biological sciences, and computer science indicates that friendships extend beyond academic similarities, possibly influenced by extracurricular activities, shared social spaces, or pre-existing relationships from high school.

The dataset was collected manually due to challenges in extracting Facebook friendship data, which may have introduced selection bias. Some students may not have included their academic profiles on Facebook, leading to incomplete representation. Expanding data collection to platforms like LinkedIn or using surveys could provide a more comprehensive picture. Further research is needed to understand why statistics students are underrepresented in the dataset and to explore additional factors influencing cross-disciplinary friendships.

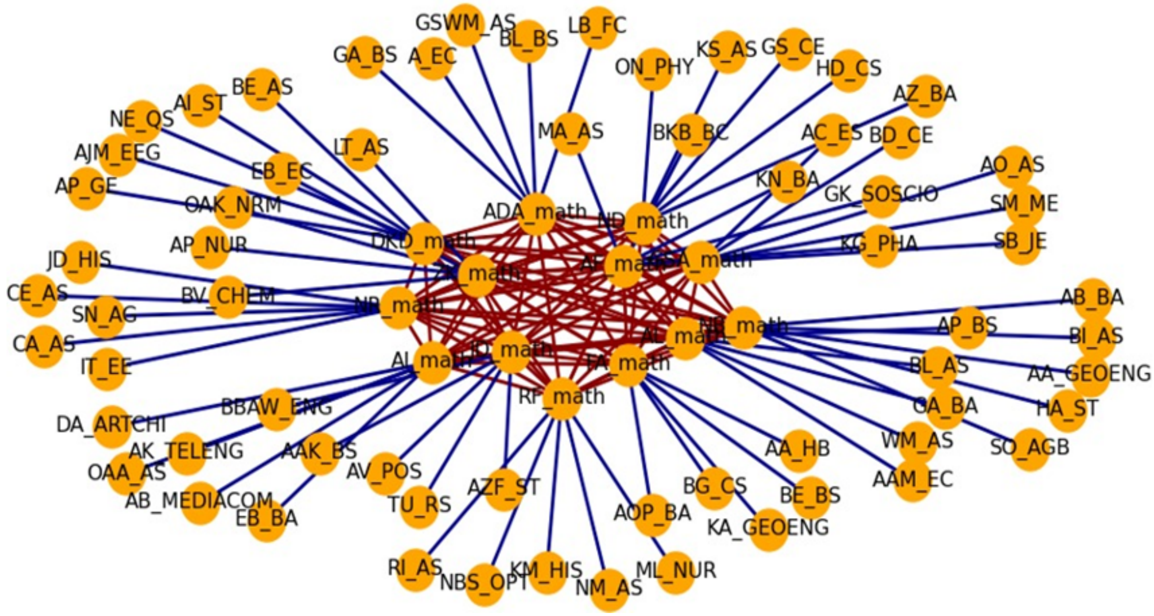

FIGURE 5 Graph final year mathematics students and their friends in other departments

Figure 5 in the article visually represents the friendship network of final-year mathematics students and their peers from other departments. The network graph consists of nodes representing students and edges representing friendships. The structure of the graph provides insights into the distribution of connections and the relative prominence of different academic programs.

A key observation from Figure 5 is the clustering of actuarial science students around mathematics students, suggesting a strong level of interconnectivity between these two groups. This confirms the numerical findings from Table 1, where actuarial science had the highest frequency of friendships. A possible explanation for this clustering is the frequent academic interactions between the two departments, as they share foundational courses, study groups, and professional interests in quantitative fields.

The graph likely shows fewer connections to students from statistics, which, despite its mathematical similarities, appears to be less integrated with mathematics students. A possible reason for this is the independent course structure of the statistics department, which might limit opportunities for frequent interactions. Another explanation could be that statistics students form stronger intra-departmental bonds rather than seeking friendships outside their field.

Connections to business administration and other departments appear more dispersed, indicating that these friendships are more scattered and less structured around a common academic foundation. This could be due to broader social interactions outside the classroom, personal interests, or past relationships, such as friendships from high school.

A conjecture based on these findings is that academic structure and shared coursework significantly influence friendship formation, leading to higher connectivity between mathematics and actuarial science students. Another conjecture is that students in highly technical or specialized fields, such as statistics or engineering, may form tighter intra-departmental bonds, reducing their likelihood of forming cross-disciplinary friendships. Finally, students in more socially diverse disciplines, such as business administration, may have more varied and dispersed connections due to their exposure to a wider range of people in non-academic settings.

Further analysis using network metrics, such as centrality measures or clustering coefficients, could provide deeper insights into the strength and influence of these connections within the overall social network.

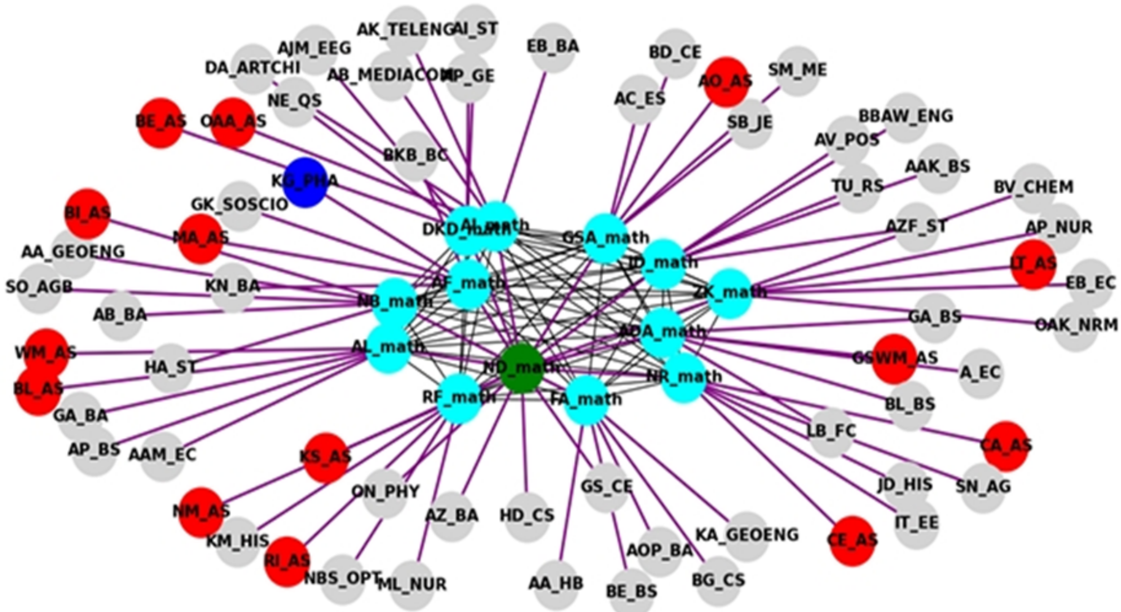

FIGURE 6 Undirected graph showing the BFS path (in purple) with starting node(green), the end node in (blue) and most common program (in red)

Figure 6 in the article presents an undirected graph illustrating the Breadth First Search (BFS) path used to analyze the friendships of final-year mathematics students. The visualization highlights the starting node in green, the end node in blue, and the most common program (actuarial science) in red. This structure allows for a clear understanding of how friendships are distributed and how different academic programs are connected within the network.

A notable feature of Figure 6 is the concentration of actuarial science (AS) students along the BFS path, confirming previous findings that mathematics students have the strongest social ties with actuarial science peers. This reinforces the idea that shared coursework, similar career paths, and frequent academic interactions facilitate these friendships. The placement of AS nodes in red suggests that they serve as key intermediaries within the network, possibly acting as bridge nodes connecting mathematics students to other disciplines.

The BFS traversal also reveals a sparse connection to statistics (ST) students, despite their academic proximity to mathematics. This suggests that while mathematics and statistics share theoretical foundations, their academic structures might encourage more intra-departmental bonding rather than interdisciplinary friendships. Another observation is the dispersed presence of business administration (BA) students, which indicates that these friendships are less clustered and may stem from broader social interactions rather than structured academic engagements.

A conjecture from Figure 6 is that mathematics students primarily form friendships within their academic or closely related fields, with actuarial science being the dominant connection due to shared coursework and professional alignment. Another possible explanation for the structure of the graph is that BFS, as a traversal method, naturally emphasizes connections based on reachability rather than overall network influence, meaning highly connected students (e.g., those with many friends) may be overrepresented. Additionally, the network structure suggests that while mathematics students interact with diverse programs, these interactions are not uniformly distributed, likely influenced by factors such as academic environment, extracurricular activities, and pre-existing social ties.

Further analysis using degree centrality, clustering coefficients, or shortest-path analysis could provide more quantitative insights into the structure of the network and how different academic programs influence social connectivity.

4 | FURTHER ANALYSIS OF RESULTS

This section outlines the results obtained from applying the BFS algorithm to the developed social network graph. It emphasizes the notable patterns identified, especially the close connections between mathematics students and actuarial science students.

4.1 | The BFS Path

$ND\_math \rightarrow ON\_PHY \rightarrow AZ\_BA \rightarrow ADAHD\_CS \rightarrow ADAKS\_AS \rightarrow ADAGS\_CE \rightarrow ADAGSA\_math \rightarrow SB\_JE \rightarrow SM\_ME \rightarrow AC\_ES \rightarrow BD\_CE \rightarrow AO\_AS \rightarrow AL\_math \rightarrow WM\_AS \rightarrow AAM\_EC \rightarrow AP\_BS \rightarrow GA\_BA \rightarrow ADABL\_AS \rightarrow ADADKD\_math \rightarrow AJM\_EEG \rightarrow NE\_QS \rightarrow ADAAP\_GE \rightarrow ADABE\_AS \rightarrow AI\_ST \rightarrow ADA\_math \rightarrow A\_EC \rightarrow ADAGSWM\_AS \rightarrow ADALB\_FC \rightarrow ADAGA\_BS \rightarrow ADABL\_BS \rightarrow ADANR\_math \rightarrow ADAIT\_EE \rightarrow ADACA\_AS\pi JD\_HIS \rightarrow ADASN\_AG \rightarrow ADACE\_AS \rightarrow ADANB\_math \rightarrow ADAAB\_BA \rightarrow ADAAA\_GEOENG \rightarrow ADASO\_AGB \rightarrow ADABI\_AS \rightarrow ADAHA\_ST \rightarrow ADAAI\_math \rightarrow ADAAK\_TELENG \rightarrow ADAAB\_MEDIACOM \rightarrow ADADA\_ARTCHI \rightarrow ADAOAA\_AS \rightarrow EB\_BA \rightarrow ZK\_math \rightarrow OAK\_NRM \rightarrow ADAAP\_NUR \rightarrow ADABV\_CHEM \rightarrow ADALT\_AS \rightarrow ADAEB\_EC \rightarrow ADAFA\_math \rightarrow ADABE\_BS \rightarrow ADAAA\_HB \rightarrow ADABG\_CS \rightarrow ADAKA\_GEOENG \rightarrow ADAAOP\_BA \rightarrow ADARF\_math \rightarrow ADAML\_NUR \rightarrow ADARI\_AS \rightarrow ADAKM\_HIS \rightarrow ADANBS\_OPT \rightarrow ADANM\_AS \rightarrow ADAJD\_math \rightarrow ADAAAK\_BS \rightarrow ADABBAW\_E \rightarrow ADAAZF\_ST \rightarrow ADATU\_RS \rightarrow ADAAV\_POS \rightarrow ADAAF\_math \rightarrow ADAKN\_BA \rightarrow ADAMA\_ASGK\_SOSCIO \rightarrow ADABKB\_BC \rightarrow ADAKG\_PHA$

The BFS path was analysed to determine the frequency of different academic programs among the friends of the mathematics students. The table below shows the program counts, indicating that actuarial science (AS) is the most common program among the friends, with 14 occurrences, followed by business administration, with a frequency of 7 and statistics with a frequency of 2.

TABLE 2 Recorded program counts from the BFS path

Program	Count
AS	14
BA	7
BS	6
CS	3
EC	2
EE	1
GEOENG	2
HIS	2
MEDIACOM	1
NRM	1
QS	1
SOCIOSO	1
ST	2

5 | DISCUSSIONS

The data presented in the Table 2 clearly shows that final year mathematics students interact most with students in the actuarial science (AS) program on Facebook, with 14 occurrences. This significant connection is due to the close relationship between the actuarial science and mathematics departments. These two departments share common courses and participate in joint lectures from the first year, leading to the formation of study groups where students benefit from the diverse perspectives offered by their respective instructors. Additionally, both departments engage in extracurricular activities such as dinners, quiz competitions, movie nights, and other social events, which contribute to a strong relationship between the two departments. As a result, it is logical that final year mathematics students have the most connections with actuarial science students on Facebook. The program with the second highest frequency turned out to be business administration, which turn out to be the most populated department, it is thereby not astonishing that final year mathematics students have a number of friends from that department. Statistics coming last was a finding that defied expectations, this is because statistics just like actuarial science has some similarities to mathematics. As a result, statistics students also have certain courses with mathematics students, which creates room for socializing among the students of the two departments. In contrast, other departments having fewer connections with final year mathematics students may have originated from their past relationships in their senior secondary school, or from meeting at denominational activities or sports-related events. Connecting mathematics students with actuarial science and statistics students is a positive idea and should be recommended due to the numerous advantages that encompass the connections.

6 | POLITICAL ECONOMY LESSONS AND IMPLICATIONS FROM THE STUDY

The study offers significant political economy insights, particularly regarding network formation, resource allocation, and the distribution of opportunities. One of the central lessons is that social networks play a fundamental role in shaping access to resources and economic mobility. The strong concentration of friendships between mathematics and actuarial science students reflects the broader reality that individuals tend to establish connections within familiar professional or academic circles. This suggests that economic opportunities and career advancement are often facilitated by existing networks rather than purely by individual merit, reinforcing the idea that social capital can be as valuable as technical skills in determining future success. The presence of dominant clusters in the study highlights how economic and professional elites form tightly connected groups, making access to high-opportunity sectors dependent on network positioning rather than open competition.

The structure of the friendship network, particularly its reliance on a select group of highly connected individuals, mirrors how information and influence spread in economic and political systems. The BFS algorithm, which identifies key nodes in the network, reflects how certain individuals or institutions control the flow of information in financial markets, policymaking, and professional advancement. Just as BFS may disproportionately highlight individuals with many connections while underrepresenting those at the periphery, economic and political systems tend to amplify the voices of well-networked actors while limiting access for those outside established circles. This has significant implications for institutional design, as it suggests that policies aimed at economic inclusion must actively seek to integrate individuals who are structurally disadvantaged by network effects.

Another key takeaway is the role of homophily in reinforcing economic stratification. The strong clustering between mathematics and actuarial science students suggests that shared professional interests and educational backgrounds create insular networks that limit exposure to broader interdisciplinary collaborations. This is consistent with the tendency in financial and corporate environments where professionals from similar academic or social backgrounds dominate decision-making spaces, creating self-reinforcing cycles of privilege. While these networks provide efficiency and trust, they also limit economic adaptability by reducing the diversity of perspectives within professional environments. Encouraging broader connections across disciplines and sectors could enhance resilience in both financial markets and policy development by integrating a wider range of ideas and expertise.

The study also suggests that network biases can lead to unequal distributions of economic benefits. The BFS algorithm, by design, emphasizes nodes with high connectivity, meaning that individuals with fewer links may remain invisible in the network analysis. In economic terms, this parallels how job markets, financial systems, and governance structures tend to favour those with preexisting influence, making it difficult for outsiders to break in. This insight is particularly relevant in understanding labour market access, where social connections often determine hiring decisions more than formal qualifications. Addressing this form of exclusion would require deliberate interventions, such as mentorship programs or alternative hiring mechanisms, to counterbalance the structural advantages enjoyed by well-connected individuals.

Moreover, the study underscores the importance of cross-sectoral collaboration and economic diversification. The limited diversity of friendships among mathematics students, with an overwhelming presence of actuarial science connections, suggests a tendency toward professional silos that may restrict broader intellectual and career opportunities. In political economy, economic specialization without diversification creates systemic vulnerabilities, as seen in financial crises where excessive interconnectivity within a single sector amplifies risks. Encouraging students to engage with a wider variety of disciplines could be compared to fostering economic policies that promote cross-industry collaboration, enhancing both resilience and innovation.

Ultimately, the study's findings illustrate how social networks shape access to economic and political resources in ways that reinforce existing hierarchies. The observed friendship patterns among students reflect broader structural dynamics in labour markets, finance, and policymaking, where connectivity influences opportunity distribution. Recognizing the economic implications of network structures highlights the need for institutional interventions that promote greater inclusion, ensuring that access to opportunities is not solely determined by preexisting social ties but is expanded to accommodate a broader range of individuals and ideas.

7 | LIMITATIONS AND FUTURE DIRECTIONS IN SOCIAL NETWORK ANALYSIS USING GRAPH THEORY

The study, while insightful in demonstrating the application of graph theory to social network analysis, has several limitations that affect the generalizability and depth of its findings. One major limitation is the data collection method, which relied on manually identifying Facebook friendships due to challenges in extracting complete and accurate academic profiles from the platform. This introduces potential selection bias, as the sample may not fully represent the broader network of final-year mathematics students. Many students either do not list their academic details on Facebook or use pseudonyms, making it difficult to ensure a comprehensive dataset. Future research could address this limitation by using more reliable and structured data sources, such as university administrative records, LinkedIn profiles, or direct surveys that ensure accurate classification of students and their academic affiliations.

Another limitation is the use of a single graph traversal algorithm (Breadth First Search - BFS), which, while effective for mapping direct connections, does not fully capture the structural complexity of the social network. BFS tends to highlight highly connected individuals while underrepresenting peripheral nodes, potentially skewing the analysis of social connectivity. Future studies could apply a comparative approach using multiple algorithms, such as Depth First Search (DFS), Dijkstra's algorithm for weighted relationships, or community detection techniques to better understand network clustering and influence.

The study also does not account for the strength of friendships beyond their existence as binary connections. Treating all friendships as equal in weight oversimplifies real-world relationships, where some connections may be stronger and more influential than others. Future research could incorporate weighted graphs that assign different levels of interaction intensity based on frequency of engagement, shared academic activities, or collaboration on projects. This would provide a more nuanced understanding of social influence within the network.

A further limitation is the narrow focus on a single academic program and institution, which restricts the applicability of the findings to broader university settings. The friendship dynamics observed among mathematics students at KNUST may differ significantly from those in other universities or among students in other disciplines. To enhance the external validity of these insights, future studies could adopt a multi-institutional approach, comparing friendship patterns across different universities, disciplines, and even geographic regions.

Lastly, the paper primarily analyzes social interactions within an academic context but does not explore how these networks influence career outcomes, knowledge-sharing, or long-term professional mobility. A longitudinal study tracking students beyond graduation could provide deeper insights into how academic friendships translate into career opportunities, entrepreneurship collaborations, or research partnerships. This would establish a clearer link between university social networks and broader economic or professional advantages.

To advance this research, future studies should leverage larger datasets, advanced network metrics, and cross-disciplinary comparisons. Integrating data from multiple social media platforms, applying machine learning techniques for network influence detection, and investigating the role of institutional policies in shaping student interactions would provide a more comprehensive and generalizable understanding of the impact of academic social networks.

8 | CONCLUSION

This paper successfully applied graph theory and the Breadth First Search (BFS) algorithm to analyze the friendship networks of final-year mathematics students at KNUST. The findings revealed that actuarial science students formed the largest cluster of connections, suggesting that shared coursework and professional alignment play a significant role in shaping social networks. Business administration students also had notable representation, indicating broader social and professional networking interests among mathematics students. Surprisingly, statistics students, despite their academic similarity to mathematics, had fewer recorded connections, pointing to possible differences in departmental structure or social interaction patterns.

The study highlights the broader implication that social networks influence access to information, academic collaboration, and potential career opportunities. The structure of friendships observed aligns with established theories in social capital and network formation, where proximity and shared interests shape relationship dynamics. However, the study also acknowledges limitations in data collection, algorithm bias, and the scope of analysis, suggesting that future research should adopt larger datasets, alternative algorithms, and a comparative approach across institutions and disciplines.

Ultimately, the research underscores the importance of understanding social networks in academic settings, not only for their immediate impact on student interactions but also for their long-term influence on professional development and economic mobility. The application of graph theory provides a valuable framework for studying these dynamics, and further refinements in methodology and data sources could yield even deeper insights into the role of social connections in education and beyond.

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How to cite this article: Amadu, Akanmike, Agyapong, Johnson & Buaka . The Political Economy of Social Networks: Analyzing Friendship Patterns among University Students Using Graph Theory, *Journal of African Political Economy and Development*, *volume 9, 2024*.